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Interstation Surface Wave Attenuation by Autoregressive Deconvolution

G.-AKIS TSELENTIS[1]

Abstract—A new technique for calculating interstation Green's functions and attenuation coefficients for seismic surface waves is presented.

The interstation Green's function is evaluated from the autocorrelation functions of the seismograms, which are obtained from a maximum entropy process.

Since a data-invariant time window is not used, the evaluated Green's function gives reliable information on both the amplitude and the phase spectra of the system.

This new technique is compared with other methods by applying them to both synthetic and real data from a path in the Canadian shield.

Key words: Attenuation, maximum entropy, surface waves.

1. Information

Useful information about the dynamic properties of the crust and upper mantle structure may be obtained from the dispersion and attenuation characteristics of surface waves, and in particular from the interstation transfer function, which is given by the spectral ratio between two waveforms observed at two stations lying on a common great circle path with the earthquake epicenter.

LANDISMAN *et al.* (1969) noticed that considerable reduction of the noise level and stabilization of the Green's function can be achieved by windowing the cross-correlogram, since the cross-correlation function is an approximation of the interstation impulse response. A different approach to the problem of calculating the interstation Green's function has been recently given by TAYLOR and TOKSÖZ (1982) and HWANG and MITCHELL (1986), which are based on the use of a time domain and a frequency domain Wiener filters respectively.

The problems that arise with conventional methods in determining interstation Green's functions are mainly due to assumptions that are made concerning the data

[1] University of Patras, Dept. of Applied Geology and Geophysics, Laboratory of Theoretical and Applied Seismology, Patras 26110, Greece.

that lie outside of the known interval (periodic or zero). In the analysis of surface wave records, whose lengths often are of the order of periods of interest, the above assumptions can become quite unreasonable and often may result in unfortunate properties of the spectral estimator. This could considerably distort the obtained phase components of the required transfer function.

In this paper we present a technique for determining the interstation Green's function following Wiener optimum filtering and autoregressive (AR) modeling. Since data-invariant time window is not used in this method, the estimated transfer function gives reliable information on both the amplitude and phase spectra of the system. Synthetic seismograms are computed using a known anelastic medium and are used to test the method. Finally, the technique is applied to a surface wave path crossing the Canadian shield.

2. *Interstation Green Function with Deconvolution*

Assume two stations positioned along the same great circle path with the earthquake epicenter (Figure 1), and letting $x(t)$ be the seismogram nearest the source. Then the record $y(t)$ at station [$S2$] can be expressed, under time-invariant system assumptions, as a convolution integral of the input $x(t)$ to the interstation crustal medium with its impulse response $h(t)$

$$y(t) = \int_0^\infty h(m)x(t-m)\,dm = h(t) * x(t). \tag{1}$$

In practice we cannot obtain a complete form of $h(m)$ for the whole interval $0 \le m < \infty$, since we use only a finite amount of data $x(n)$, $y(n)$ for the interval $0 \le n \le N$. The error difference between the observed and estimated waveforms is given by

$$S(n) = y(n) - x(n-m)h(m). \tag{2}$$

By considering the spectra of the recorded seismograms and the required impulse response (transfer or Green's function) then eq. (1) can be written in the frequency domain as follows

$$Y(f) = X(f)H(f) \tag{3}$$

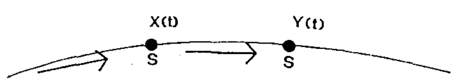


Figure 1
Schematic illustration of the interstation Green's function.

and an obvious solution to the deconvolution problem is

$$H(f) = Y(f)/X(f). \tag{4}$$

In practice, this straightforward division of spectral terms will lead to erroneous results if the noise in $Y(f)$ has significant power at those frequencies where $X(f)$ is small. Since the deconvolution can be very unstable because of random noise, multipathing and interference by other modes, statistical stability and meaningful results can be obtained only if eq. (4) is altered, and a variety of deconvolution techniques have been suggested to accomplish this goal (OLDENBURG, 1981; SILVIA and ROBINSON, 1979).

Among these deconvolution schemes, of particular use has been the Wiener method (PEACOCK and TREITEL, 1969) which has been employed by TAYLOR and TOKSÖZ (1982) for determining the interstation Green's function from surface wave measurements.

The basic principle of Wiener deconvolution is mean square optimization which implies that the expected value of the squared difference between the desired and actual output is minimized

$$\{Sm\}^2 = E(h_0, h_1, \ldots, h_N) = \text{minimum}. \tag{5}$$

The minimum of $E(h_0, h_1, \ldots, h_N)$ corresponds to a point (in the $N+1$ dimensional space of the coefficients), where the partial derivatives of E, with respect to the coefficients vanish and we obtain the following system of equations

$$[R][H]^T = [C]^T \tag{6}$$

where $[R]$ is the matrix containing the coefficients of the autocorrelation of the input x_t, $[H]^T$ is a column vector containing the required interstation impulse response, and $[C]^T$ is a column vector containing the cross-correlation between input and output.

The structure of the matrix $[R]$ (Toeplitz form), makes possible the application of a very efficient iterative method, due to LEVINSON (1974). In order to obtain stable solutions of eq. (6), sometimes the principal diagonal of the matrix $[R]$ is modified by adding a small constant to its elements.

In the frequency domain, the deconvolution operation can be considerably simplified since the normal equation (6) can be written as

$$C(f) = H(f)R(f) \tag{7}$$

where $C(f)$, $H(f)$ and $R(f)$ are the Fourier transforms of the cross-correlation, Green's function, and autocorrelation, respectively. The required Green's function can be easily obtained as the ratio of the smoothed cross-spectrum to the smoothed autospectrum

$$H(f) = C(f)/R(f). \tag{8}$$

From the above it is obvious that, if the autocorrelation $[R]$ and cross-correlation $[C]$ matrices are given, the required Green's function operator $[H]$ can be determined either from eqs. (6) or (8).

The apparent advantage of Wiener filtering over the use of simple spectra ratios (eq. 4), is the windowing which cuts off the noise outside the window. Furthermore, HWANG and MITCHELL (1986), noting that the autocorrelation function usually has a shorter duration, improved the Wiener filtering by applying a shorter window in the autocorrelation function to achieve a larger reduction of noise.

Since the derivation of both the autocorrelation and cross-correlation functions are expressed as infinite time series of the input x_t and the output y_t, and since x_t, y_t are given only within a finite interval, the assumptions that we make about the data beyond the end points, inevitably introduce an artificial phase shift and make spectral resolutions worse.

In order to avoid these difficulties, we shall follow Burg's method and develop a procedure of calculating the Green's function operator $[H]$. The main point is that Burg's iterative algorithm is used to calculate the correlation operators and the medium impulse response.

3. *Formulation*

The details of Burg's method have been described by many authors (e.g., ULRYCH and BISHOP, 1975; ULRYCH and CLAYTON, 1976; KAY, 1988). In the following, we shall give only a brief description of the formulae required.

Let the prediction error operator for x be $a[m]$, $(0 \le m \le M)$ where $a[0] = 1$. Then the forward linear prediction estimate is given by

$$e^f_M[n] = x_t[n] + \sum_{m=1}^{M} x_t[n-m]a_M[m] \tag{9}$$

and the backward linear prediction error by

$$e^b_M[n] = x_t[n-m] + \sum_{m=1}^{M} x_t[n+m-M]a^*_M[m] \tag{10}$$

where * stands for complex conjugate.

It is known (e.g., KAY, 1988; ULRYCH and BISHOP, 1975), that the Levinson recursive solution to the Yule-Walker equations relates the autoregressive (AR) parameters of order M to those of order $M-1$ as

$$a[m] = a_{M-1}[m] + K_{M-1}a^*_{M-1}[M-m] \tag{11}$$

for $m = 0$ to M, where K is the reflection coefficient.

Substituting eq. (11) into eqs. (9) and (10), we obtain the following recurrence relations (MARPLE, 1987)

$$\begin{aligned} e^f_M[n] &= e_{M-1}[n] + K_{M-1}e_{M-1}[n-1] \\ e^b_M[n] &= e_{M-1}[n-1] + K_{M-1}e_{M-1}[n]. \end{aligned} \tag{12}$$

By minimizing the sum of squares of the forward and backward prediction errors over the time interval $0 < n < N$

$$q^{fb}_M = \sum_{n=0}^{N} \{[e^f_M[n]]^2 + [e^b_M[n]]^2\} = \text{minimum}$$

we can calculate the reflection coefficient. It is important to note that the summation ranges only over the available data. Thus, q^{fb}_M is a function of a single parameter K since the prediction errors from order $M-1$ will be known.

Differentiating q^{fb}_M with respect to the real and imaginary parts of K and setting the result equal to zero, yields

$$\partial q/\partial \operatorname{Re}\{K_{M-1}\} + i\partial q/\partial \operatorname{Im}\{K_{M-1}\} = 0 \tag{13}$$

and solving for K_{M-1} we get

$$K_{M-1} = \left\{-2\sum_{n=0}^{N} e^f_{M-1}[n]e^b_{M-1}[n-1]\right\} \bigg/ \left\{\sum_{n=0}^{N} [e^b_{M-1}[n-1]^2 + e^f_{M-1}[n]]^2\right\}. \tag{14}$$

In order to simplify the evaluation of the above expression, we used the following recursion proposed by ANDERSON (1978)

$$D_M = (1 - |K_{M-1}|^2)D_{M-1} - |e^f_{M-1}[M]|^2 - |e^b_{M-1}[N]|^2 \tag{15}$$

where D is the denominator in eq. (14).

After determining the reflection coefficients from equations (12)–(15) we can determine a, e^f_M, e^b_M, and the autocorrelation from the quantities of $(M-1)$-th order.

Following a similar iterative process we can evaluate the interstation impulse response h. Comparing a set of equation (6) of the M-th order with that of the $(M-1)$-th order we obtain

$$h_M[m] = h_{M-1}[m] + L_{M-1}a_M(M-m) \tag{16}$$

for $0 \leq m \leq M$ and $h_{M-1}[M] = 0$ while L is the corresponding reflection coefficient as above.

Thus, eq. (2) yields

$$S_M[n] = S_{M-1}[n] - L_{M-1}e^b_M[n] \tag{17}$$

by minimizing the sum of squares $S[n]$ over the time interval $0 \leq n \leq N$ in a similar way as above we have

$$L_{M-1} = \left\{ \sum_{n=0}^{N} S_{M-1}[n] e_M^b[n] \right\} \Big/ \left\{ \sum_{n=0}^{N} e_M^b[n]^2 \right\} \tag{18}$$

and by combining the two iterative processes derived above with the following initial conditions

$$\begin{gathered} R[0] = \sum_{n=0}^{N} x_t[n], \quad C[0] = \sum_{n=0}^{N} x_t[n] y_t[n], \quad a_0[0] = 1 \\ h_0[0] = C[0]/R[0] \\ e^f[n] = e^b[n] = x_t[n] \\ S_0[n] = y_t[n] - h_0[0] x_t[n], \quad n = 0, 1, \ldots, N \end{gathered} \tag{19}$$

we can evaluate the required interstation impulse response.

In general, the above procedure produces accurate AR estimates for the data which are truly AR (NUTTALL, 1976). For sinusoidal data, however, some bias in the estimate may occur (CHEN and STEGEN, 1974; KAVEH and LIPPERT, 1983). Although significant bias has not been observed during the present analysis, we have incorporated into the processing scheme the following data, adapting a weighting procedure proposed by HELME and NIKIAS (1985)

$$w_{M-1}[n] = \sum_{k=n-M+1}^{n-1} |x[K]| \quad \text{for} \quad M \geq 2 \tag{20}$$

which represents the common data energy in the forward and backward linear prediction errors $e_M^f[n]$ and $e_M^b[n-1]$ at time index n.

4. *Application to Synthetic Seismograms*

Consider a dispersed and attenuated wave registered at stations $S1$, $S2$ (Figure 1), separated a distance r apart. If $A1(\omega)$ is the wave amplitude at station $S1$, then the following expression obviously describes the wave at station $S2$

$$\begin{aligned} f(t, r) &= (1/2\pi) \int_{-\infty}^{\infty} A2(\omega) \exp[i(\omega t - kr)]\, d\omega \\ &= (1/2\pi) \int_{-\infty}^{\infty} \{A1(\omega) \exp -[a(\omega) + i\omega/c(\omega)]r\} \exp(i\omega t)\, d\omega \end{aligned} \tag{21}$$

where $a(\omega)$ is the attenuation parameter and $c(\omega)$ is the phase velocity.

Expressing eq. (21) in terms of the quality factor Q and frequency f, we obtain

$$f(t, r) = \int_{-\infty}^{\infty} \{A1(f)\exp-[\pi f/Q(f)u(f) + i2\pi f/c(f)]r\}\exp(i2\pi ft)\, df \quad (22)$$

where $u(f)$ is the group velocity. This equation was used to generate synthetic waves in order to verify the previously described computational approach.

For simplicity, a one-cycle triangle wave with a period of 20 sec (Figure 3a), was dispersed according to the group and phase velocity curves of Figure 2. These curves were constructed by fitting the 7th degree polynomials to surface wave dispersion data from the Canadian shield (e.g., DEAN, 1986).

The obtained waves at distances of 2500 km and 500 km from the source and for an attenuating medium of constant $Q = 200$ are shown in Figures 3b and 3c, respectively.

The corresponding amplitude spectra, as were obtained from conventional FFT analysis and from the previously described AR analysis, are shown in Figures 4a, b and Figures 5a, b for stations $S1$ and $S2$, respectively.

The AR spectra were calculated assuming three different model orders. It is obvious that too low an order resulted in a smoother estimate, while too large an order increased the resolution and introduced some spurious details into the spectrum.

Next, an attempt was made to recover the Q value from synthesized waveforms at $S1$ and $S2$, using the formula

$$Q = [\pi fr/u(f)]/[\ln(H(f))] \quad (23)$$

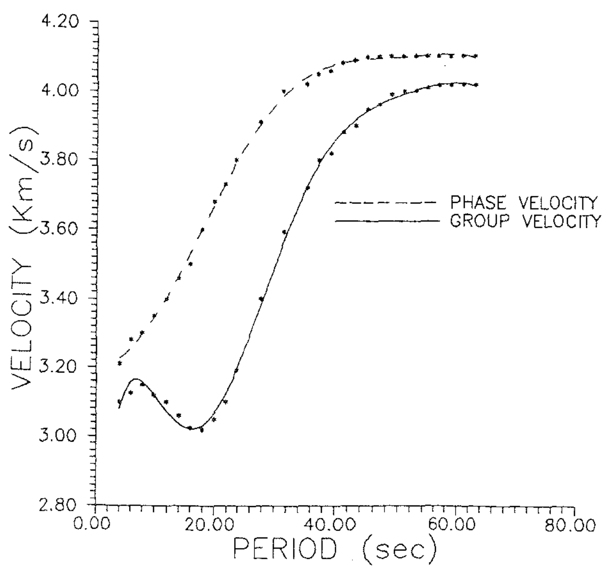
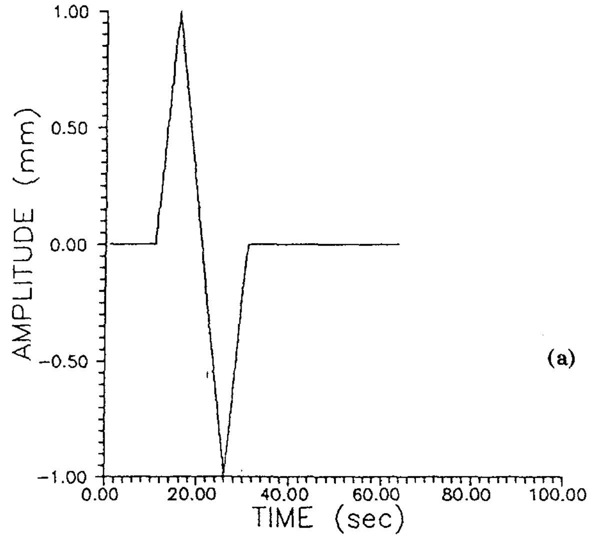


Figure 2

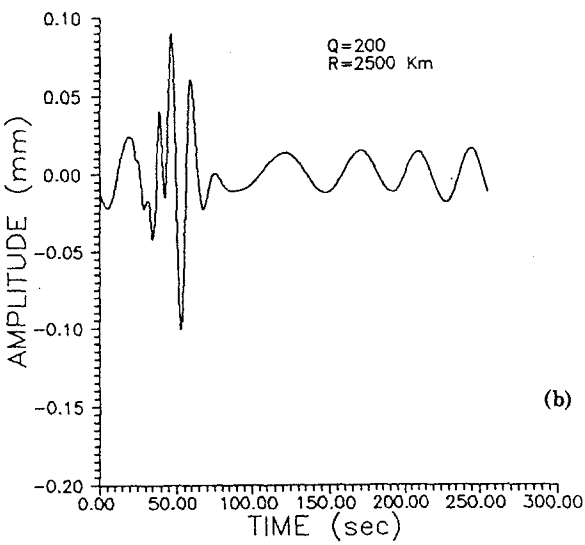
Group and phase velocity curves for the Canadian shield used in the present work.

where $u(f)$ is the group velocity and $H(f)$ is the amplitude spectrum of the interstation impulse response.

Gaussian random noise with zero mean and a standard deviation of 10% of mean absolute amplitude of the synthesized signals were added and Green's functions were calculated from (a) Fourier spectral division, (b) AR spectral division, (c) Wiener deconvolution and (d) AR deconvolution.



(a)



(b)

Figure 3(a, b)

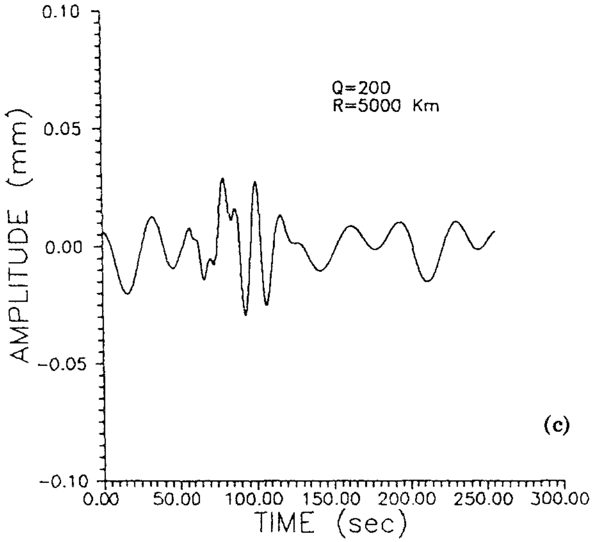


Figure 3
(a) Nondispersed wavelet, (b), (c) dispersed wavelets.

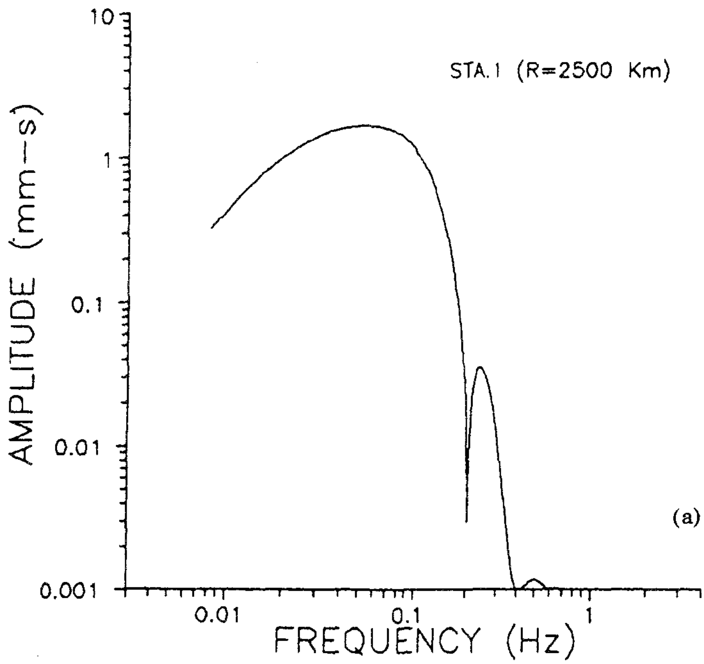


Figure 4(a)

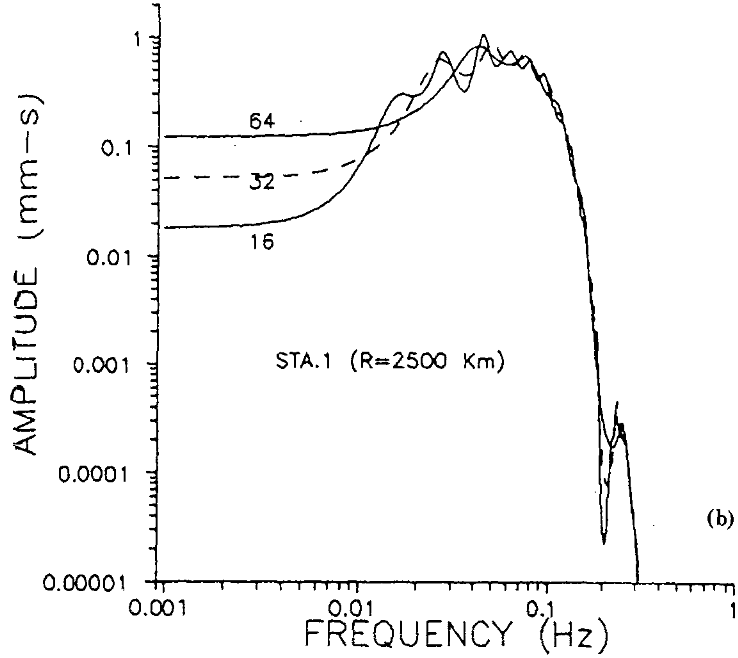


Figure 4
(a) Fourier amplitude spectra, (b) MEM amplitude spectra at station 1, with three model orders.

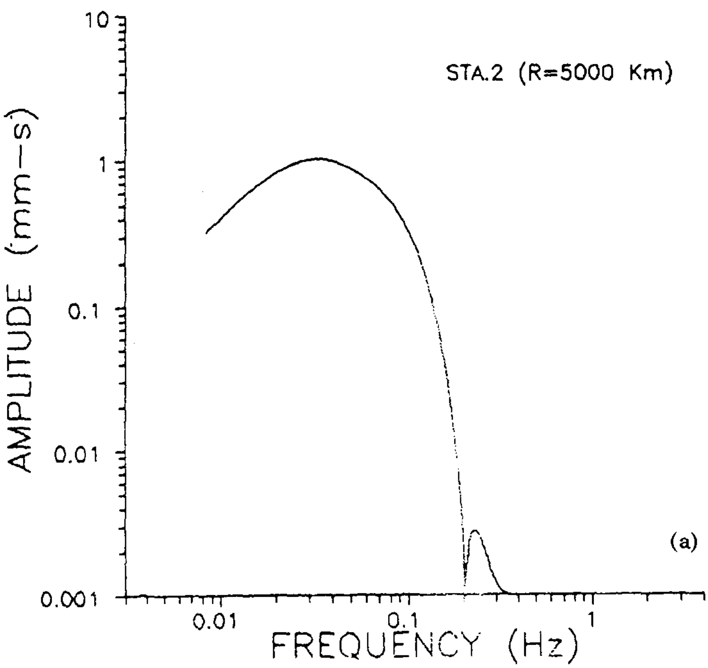


Figure 5(a)

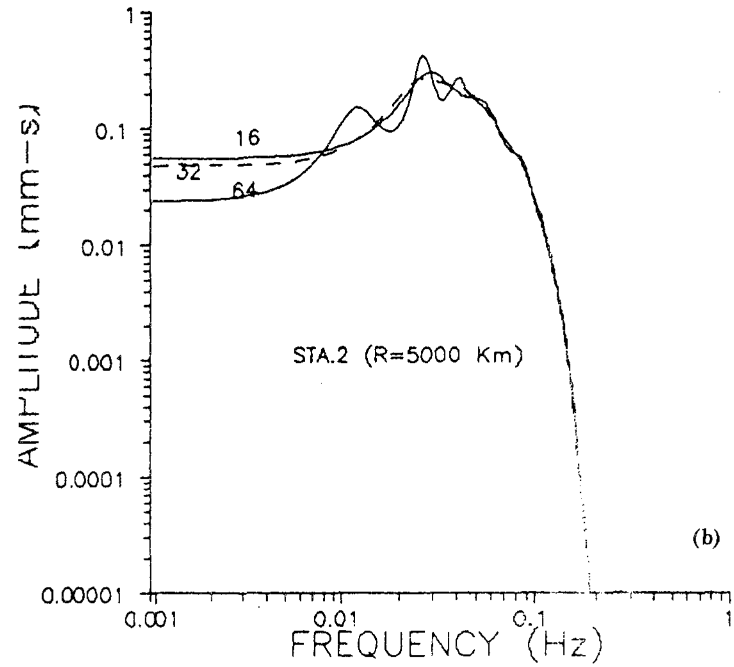


Figure 5
(a) Fourier amplitude spectra, (b) MEM amplitude spectra at station 2, with three model orders.

In the last case, the correlation functions of the synthesized seismograms were calculated from the previously described algorithm. Both Wiener deconvolution and AR deconvolution recovered successfully the initially assumed Q value of 200. This is clearly shown in Figure 6.

Fourier amplitude spectral division resulted in considerable scatter of the data, due to the noise, and we were not able to recover the initially assumed Q value. On the other hand, AR spectral division resulted in a relatively stable determination of Q values (Figure 7), particularly for the lower models. The higher orders tend to oscillate around a mean value which was the correct Q value. This result is not surprising since an AR process is equivalent to a zero-lag inverse filter and the instabilities due to noise are being reduced during the application of the filter.

Figure 7 suggests that we might be able to recover Q directly from AR spectral ratios by averaging the results obtained from lower order models.

The corresponding auto- and cross-correlation functions are shown in Figure 8.

5. Application to Real Data

The proposed technique was applied to a surface wave path from an event occurring under Baffin Island, Canada on 2 December 1970 (11: 03: 09.8;

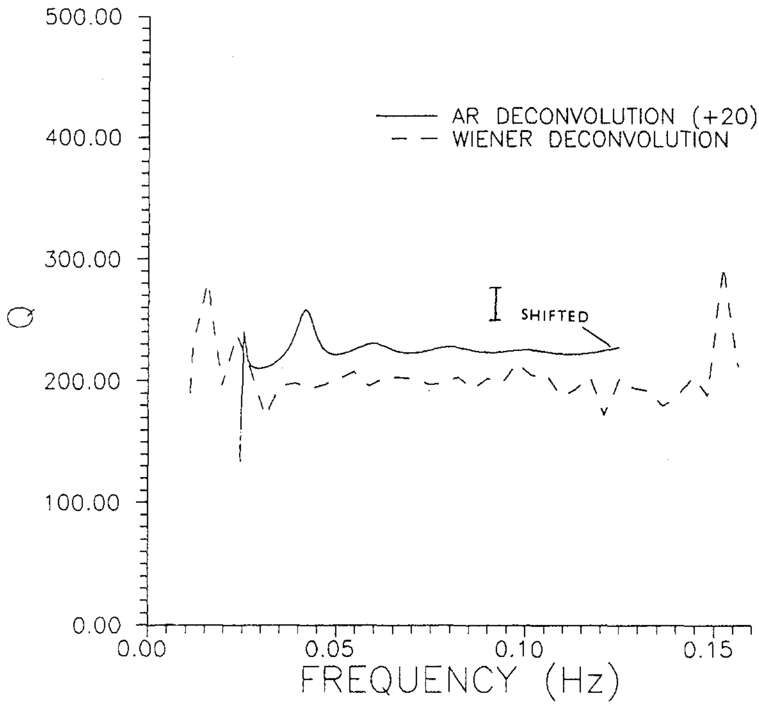


Figure 6
Comparison between Wiener and AR deconvolution.

68.4N,67.4W, $m = 4.9$, depth = 8.6 km), crossing the Canadian shield between stations SCH and HAL of the Canadian seismic network (Figure 9).

Figures 10a, b show the transverse component seismograms of the two stations (after deconvolving the instrument). Prior to processing, the digitized data were passed through a Butterworth bandpass filter with frequency range 0.02–0.15 Hz (in forward and reverse directions to avoid phase shift).

Figure 11, shows the obtained results from AR spectral ratios and Wiener deconvolution, while Figure 12 compares the results from AR deconvolution, Wiener deconvolution and average AR12, AR16 and AR20 spectral ratios. The obtained interstation Green's function is shown in Figure 13. Judging from these figures it is obvious that both AR deconvolution and AR averaging provided relatively smooth estimates of Q which compare favorably with the Q-values obtained from Wiener (spectral) deconvolution.

6. *Conclusions*

In this paper we have applied an autoregressive approach to the problem of determining the interstation attenuation parameters from surface waves.

A normal equation for an estimator of the interstation Green's function was

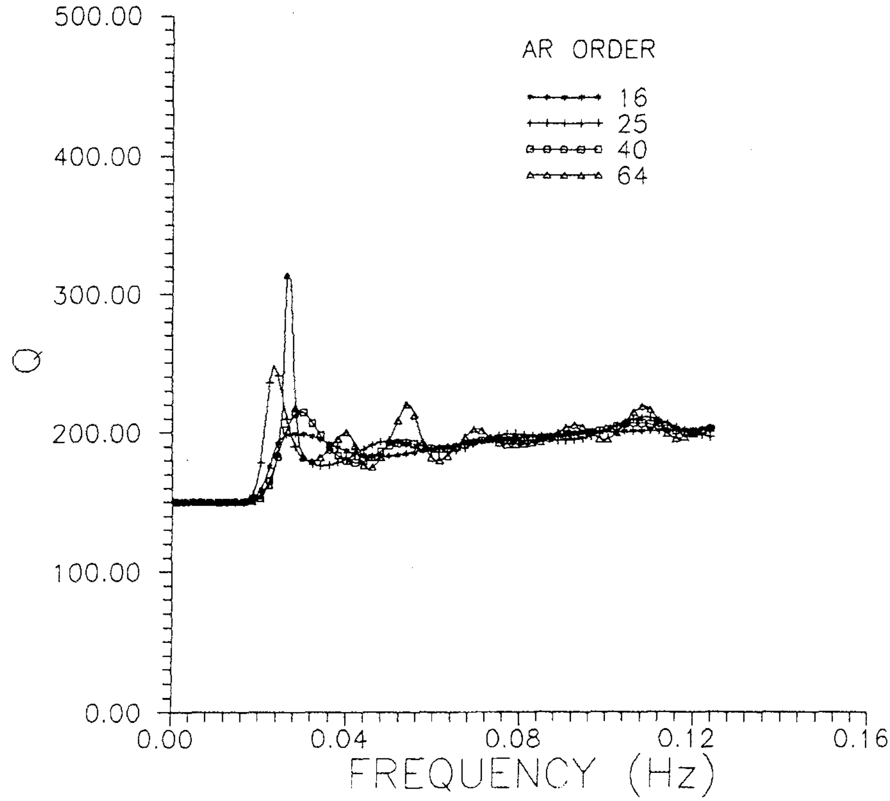


Figure 7
Comparison between AR spectral ratios for various model orders.

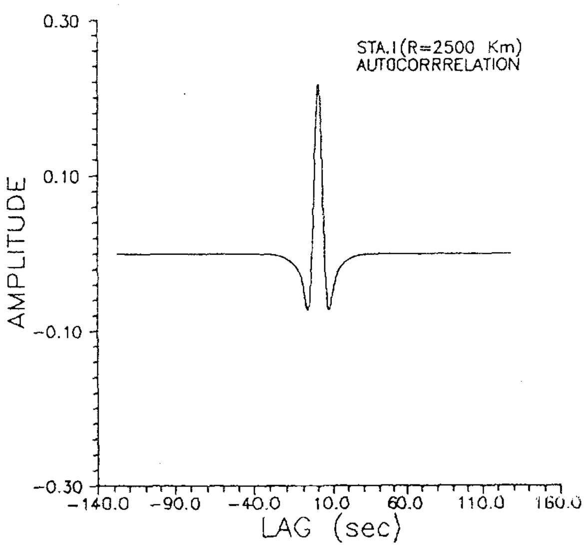


Figure 8(a)

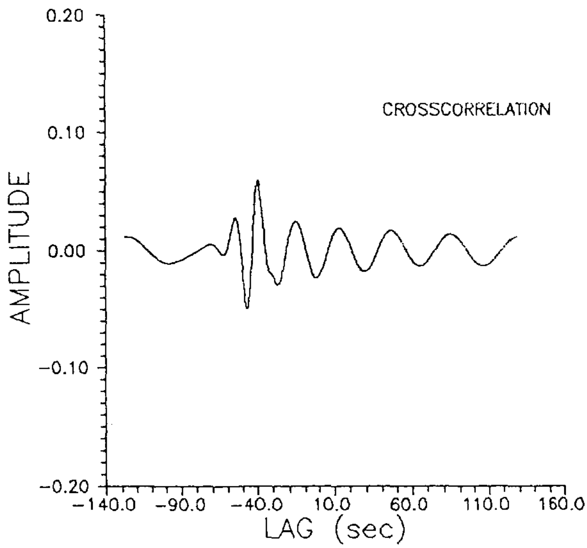


Figure 8
(a) Autocorrelation and (b) cross-correlation functions.

derived according to Wiener's optimum theory, and it was solved using a processing scheme based on Burg's algorithm.

Tests conducted using synthesized and real data showed that the technique gives results similar to those obtained from Wiener deconvolution. The analysis showed also that after a careful selection of the orders of the AR model, it is possible to obtain reliable interstation transfer functions by simply averaging the lower order AR spectral ratios. Since no data-invariant time window is used, the estimated interstation Green's function gives us significant information on both the amplitude and the phase spectra of the system.

All the software was implemented on an IBM AT computer.

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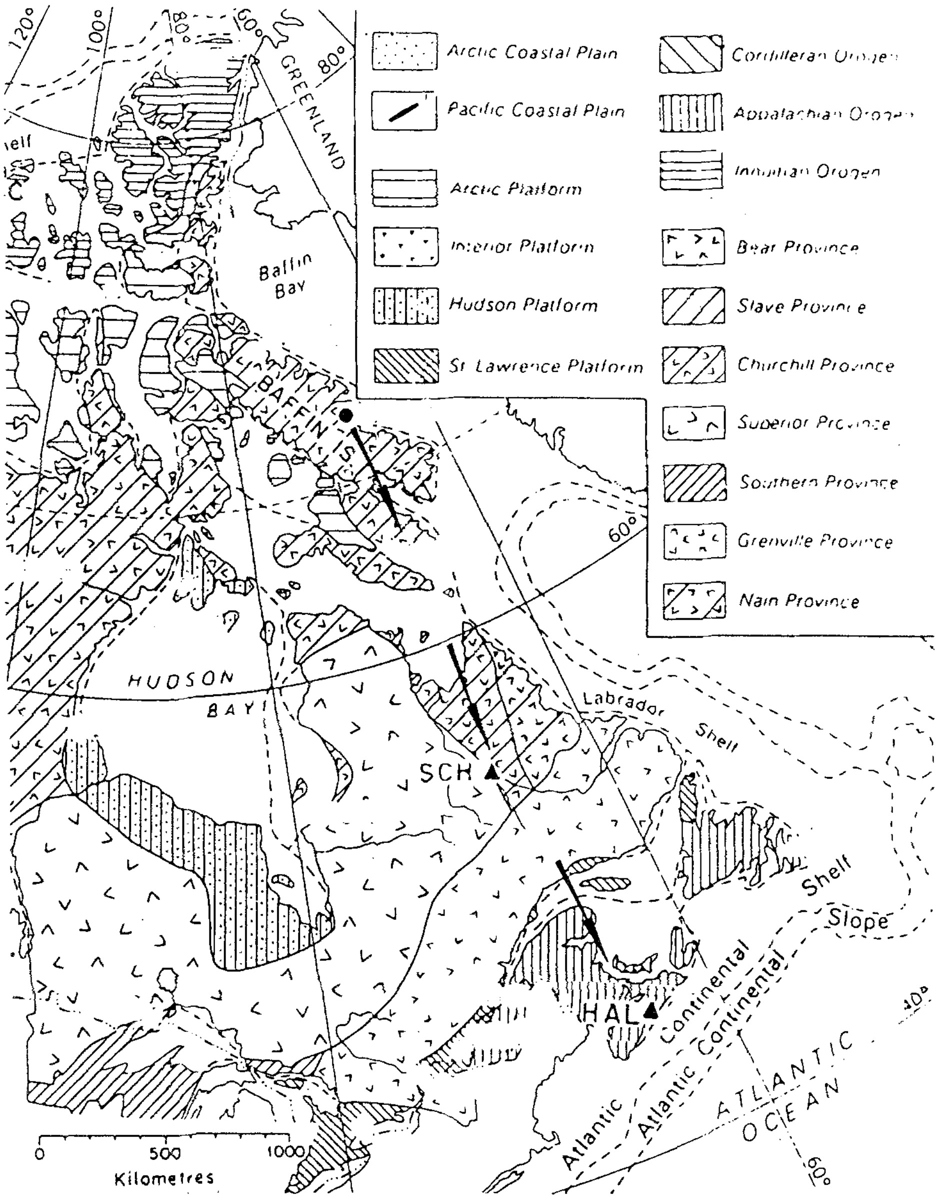

Figure 9
Location of the earthquake and stations used, superimposed on the regional geology of the area.

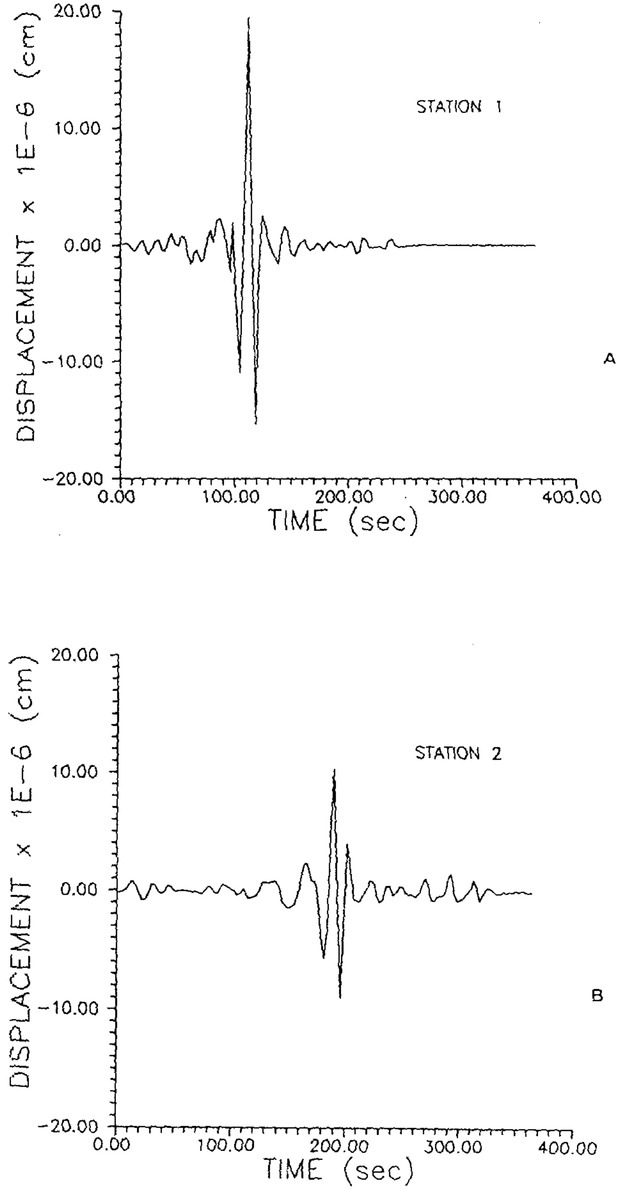

Figure 10
Transverse component seismograms for the 2 December 1970 earthquake recorded at SCH and HAL.

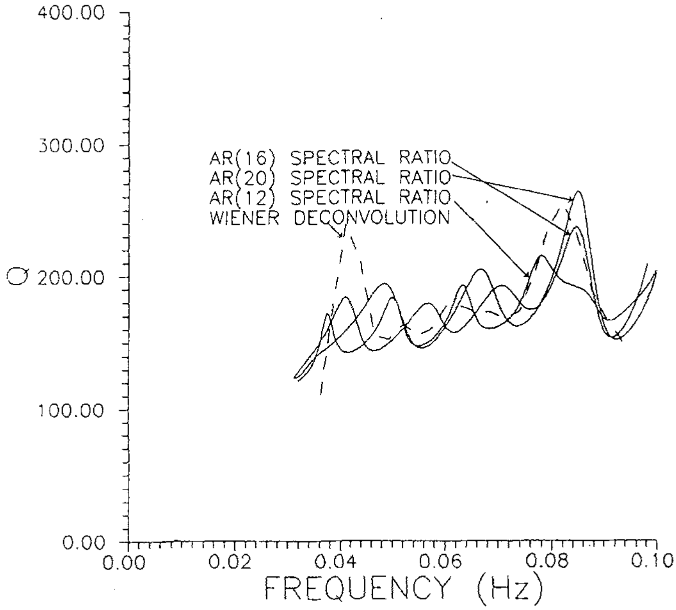

Figure 11
Comparison of the Q-values obtained from Wiener deconvolution, and different AR order spectral ratios.

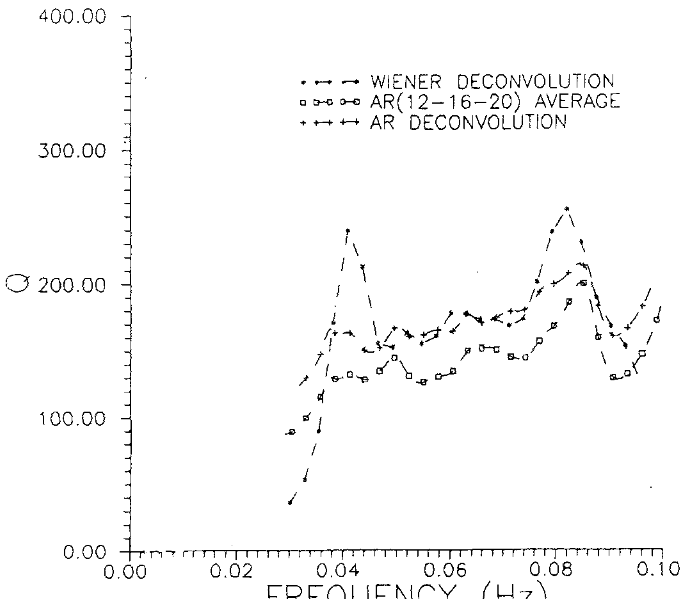

Figure 12
Comparison of the Q-values obtained from Wiener deconvolution, AR deconvolution and the average of AR spectral ratios.

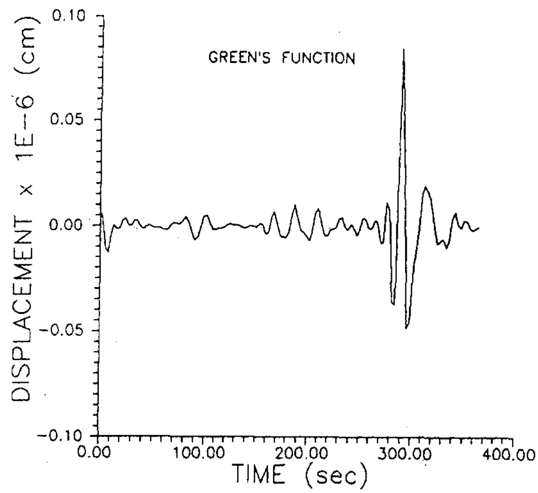

Figure 13
Interstation Green's function.

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